

**Influence of temperature and composition on some physical
properties of milk and milk concentrates. II. Viscosity**

By F. FERNÁNDEZ-MARTÍN

Influence of temperature and composition on some physical properties of milk and milk concentrates. II. Viscosity

By F. FERNÁNDEZ-MARTÍN

*Instituto de Productos Lácteos y Derivados Grasos, Patronato
'Juan de la Cierva', C.S.I.C., Madrid, Spain*

SUMMARY. Ten different types of milk, arranged in 3 sets, were observed to be pseudo-plastic fluids with linear behaviour at high shear rates in a coaxial cylinder rheometer; at low rates of shear all the samples could be considered sufficiently Newtonian and a Höppler viscometer was used for their study. Dynamic viscosity coefficients of skim-milk, half-and-half milk, whole milk and their concentrates up to about 30 % total solids were determined every 5 °C between 0 and 80 °C. For every milk type a quadratic equation relating t to $\log \eta$ was obtained. At a given temperature the concentration dependence of η for the 3 milk sets was expressed by a single relation of the same mathematical model as in the previous case. A general expression was derived to enable η to be calculated at any temperature (0–80 °C), and for any concentration (0–30 % total solids) whatever the fat to solids-not-fat ratio (0.01–0.4). A nomogram was also constructed for the immediate computation of η without any practical loss of accuracy.

It is well known that the rheological behaviour of milk products is very complex, depending mainly on temperature and concentration and physical state of their disperse phases. The physical state of fat and proteins may be affected by several factors including the experimental circumstances under which the observation is made; among them, thermal and mechanical treatments, pH and ageing have been dealt with in several papers, but only qualitative agreement exists between the results of the several workers. Many authors have proposed viscosity-temperature relationships based on the equations of Guzmán-Andrade, Poiseuille or Cox, but according to Cox (1952) in his examination of the available literature, a consistent general relationship has not been obtained. With respect to composition, at a given temperature, milk viscosity increases either with increasing fat content when solids-not-fat (SNF) content is kept constant, or with increasing SNF content for constant fat percentage; there is general agreement that milk viscosity is a non-linear function of total solids content.

An up-to-date review of the literature showed that only one attempt had been made to establish a mathematical relationship between milk viscosity, temperature and concentration: Torssell, Sandberg & Thureson (1949) studied the viscosity of 2 sets of concentrates (skim-milk and pasteurized milk containing 3 % fat) up to 40 % total

solids in the range 20–60 °C. They found an empirical Walther-type equation as follows:

$$\log \log(v_t + 0.8) = A.s + B - (C.s + D) \log \frac{T}{313} \quad (1)$$

relating the kinematic viscosity v (cS), with total solids content s (% w/w) and temperature T (°K), and where the constants, A , B , C and D had specific values for each set of milks. Different values for v_t are computed from each expression when $s = 0$, none of them fitting the theoretical values for water.

The present work deals with viscosity measurements on different types of milk and their concentrates over a wide temperature range, in an attempt to find general analytical expressions of practical value for heat and mass transfer phenomena in liquid milks (Fernández-Martín & Montes, 1970; Fernández-Martín, 1972).

EXPERIMENTAL

Materials

Commercial sterilized and, when applicable, homogenized samples were employed with the exception of concentrates of skim- and half-and-half milk, which were prepared in a laboratory rotary evaporator at about 40–50 °C under water-pump vacuum, and type W-1.5 which was reconstituted from W-1 and W-2 (v/v). Fat (f) and total solids (s) were determined by the Gerber and desiccation methods respectively; in addition nitrogen was determined by the Kjeldahl method, homogenization efficiency by the Farrall Index, and pH was measured at 25 °C. The characteristics of the samples are given in Table 1.

In order to standardize the experimental conditions, all samples were submitted to the same mechanical and thermal treatments. The samples were homogenized in a laboratory machine (Ormerod Engineers, type QP homogenizer) by recirculating them for 15 min at about 45 °C and 1000 lb/in²; they were then degassed under water-pump vacuum, poured into the viscometer tube, which was maintained at 0 °C by a thermostat, and kept there overnight as the crystallization period.

Table 1. *Analytical characteristics of the samples (N = 5)*

Milk series	Concentration grade, ϕ	Fat content (f), %		Total solids (s), %		Nitrogen, mean %	Farrall index	pH at 25 °C
		Mean	Range	Mean	Range			
Skim-milk, S	1	0.11	0.07–0.17	8.36	8.14–8.72	0.46	—	6.6
	2	0.18	0.17–0.19	16.62	16.30–16.98	0.91	—	6.5
	3.4	0.35	0.32–0.36	28.31	28.01–28.56	1.50	—	6.3
Half-and-half milk, H	1	1.56	1.50–1.64	9.17	9.15–9.22	0.44	0.5	6.2
	2	3.05	2.99–3.07	18.53	17.77–18.93	0.90	5–10	6.1
	3.2	4.87	4.78–5.10	29.07	28.59–29.64	1.40	5–10	6.1
Whole milk, W	1	3.05	3.01–3.09	11.03	10.86–11.25	0.46	0	6.5
	1.5	4.58	4.50–4.63	16.43	16.19–16.59	0.68	0	6.3
	2	6.01	5.97–6.09	21.96	21.70–22.40	0.92	0	6.1
	2.5	7.28	7.21–7.36	27.47	26.97–27.65	1.18	0	6.1

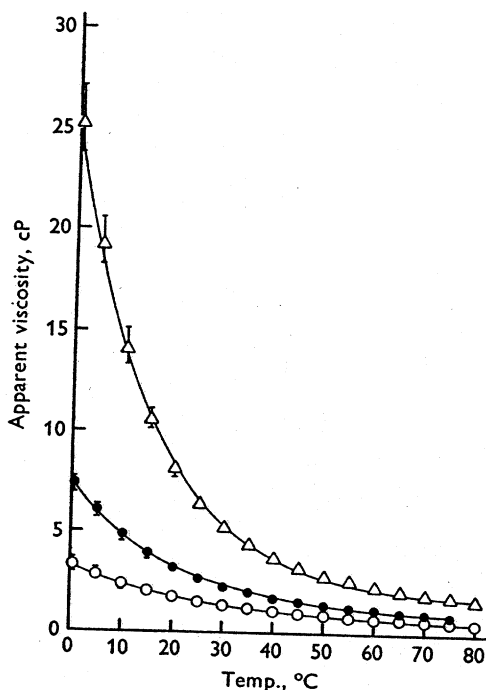


Fig. 1. Effect of temperature on the viscosity of the skim-milks. Range and mean values of 5 samples for each concentration. \circ , S-1; \bullet , S-2; Δ , S-3-4; full lines, $\log \eta = \sum_0^2 a_i t^i$.

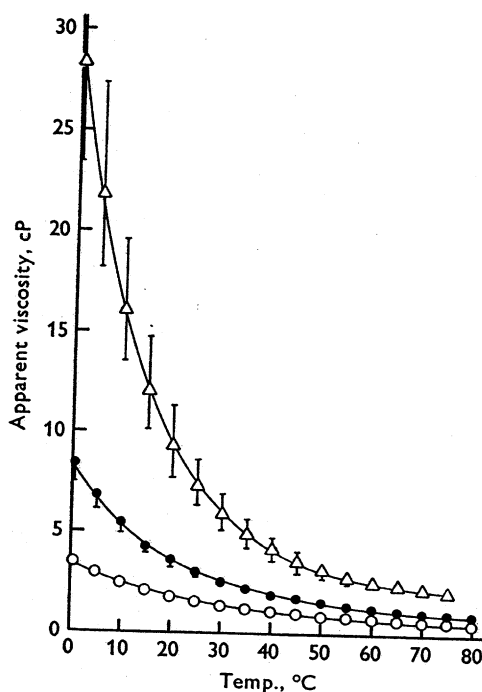


Fig. 2. Effect of temperature on the viscosity of the half-and-half milks. Range and mean values of 5 samples for each concentration. \circ , H-1; \bullet , H-2; Δ , H-3-2; full lines, $\log \eta = \sum_0^2 a_i t^i$.

Method

Apparent viscosities were determined by means of a rolling-ball, Höppler-type viscometer provided with a temperature-controlled jacket through which liquid controlled to $\pm 0.05^\circ\text{C}$ was pumped. The times for fall and half-fall were measured in 4 consecutive runs by means of a 1/10 s stop-watch and the time averaged. The density of the samples was computed from dilatometric measurements.

In previous experiments, all the samples were tested in a Drage coaxial cylinder viscometer (Rheomat 15) and found to be slightly pseudoplastic: shear-thinning effects decreased with increasing temperature and with decreasing total solids. Thus, unconcentrated milks behaved practically as Newtonian fluids throughout the range of shear rates in the absence of turbulence. Dependence on shear was shown more clearly by concentrated milks; assuming a power-law relation between viscosity and shear-rate, the exponent ranged between 0.9 and 1 so that, in practice, these milks could also be considered Newtonian. The Höppler viscometer therefore seemed fairly applicable for this study, and its hermetic seal and dilatation chamber were added advantages.

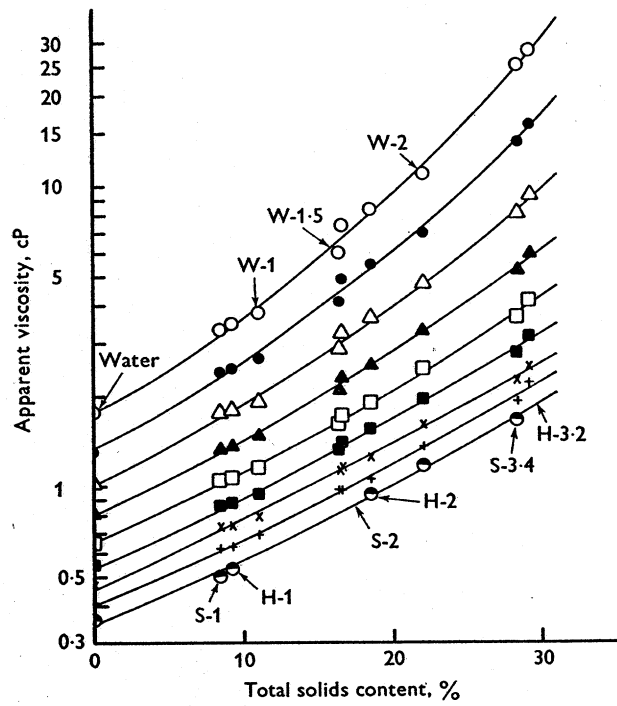


Fig. 4. Concentration effects of the 3 sets of milks showing the temperature dependence. \circ , 0.5 °C; \bullet , 10 °C; \triangle , 20 °C; \blacktriangle , 30 °C; \square , 40 °C; \blacksquare , 50 °C; \times , 60 °C; $+$, 70 °C; \odot , 80 °C; full lines, $\log \eta = \sum_{i=0}^2 b_i s^i$. Only 1 of every 2 isotherms has been plotted for the sake of clarity.

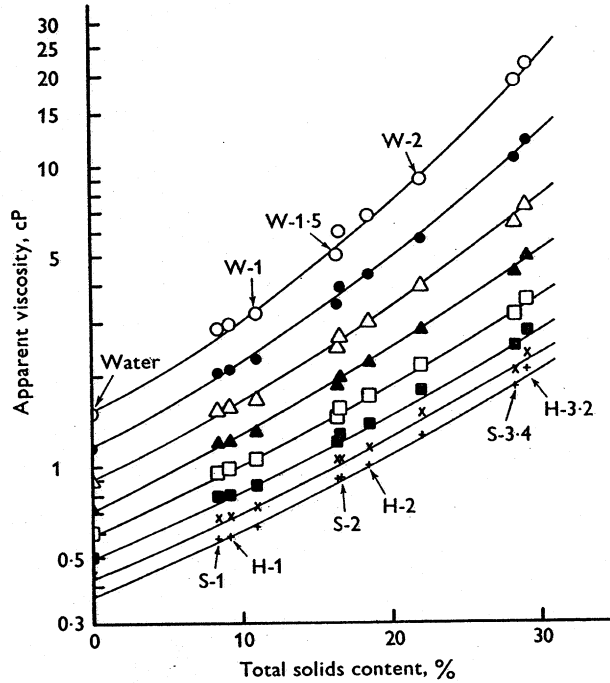


Fig. 5. Plots of the relation $\log \eta = \sum_{i=0}^2 A_i t^i + \sum_{i=0}^2 B_i t^i s + \sum_{i=0}^2 C_i t^i s^2$ at different temperatures (other than those of Fig. 4). Experimental data represented by points: \circ , 5 °C; \bullet , 15 °C; \triangle , 25 °C; \blacktriangle , 35 °C; \square , 45 °C; \blacksquare , 55 °C; \times , 65 °C; $+$, 75 °C.

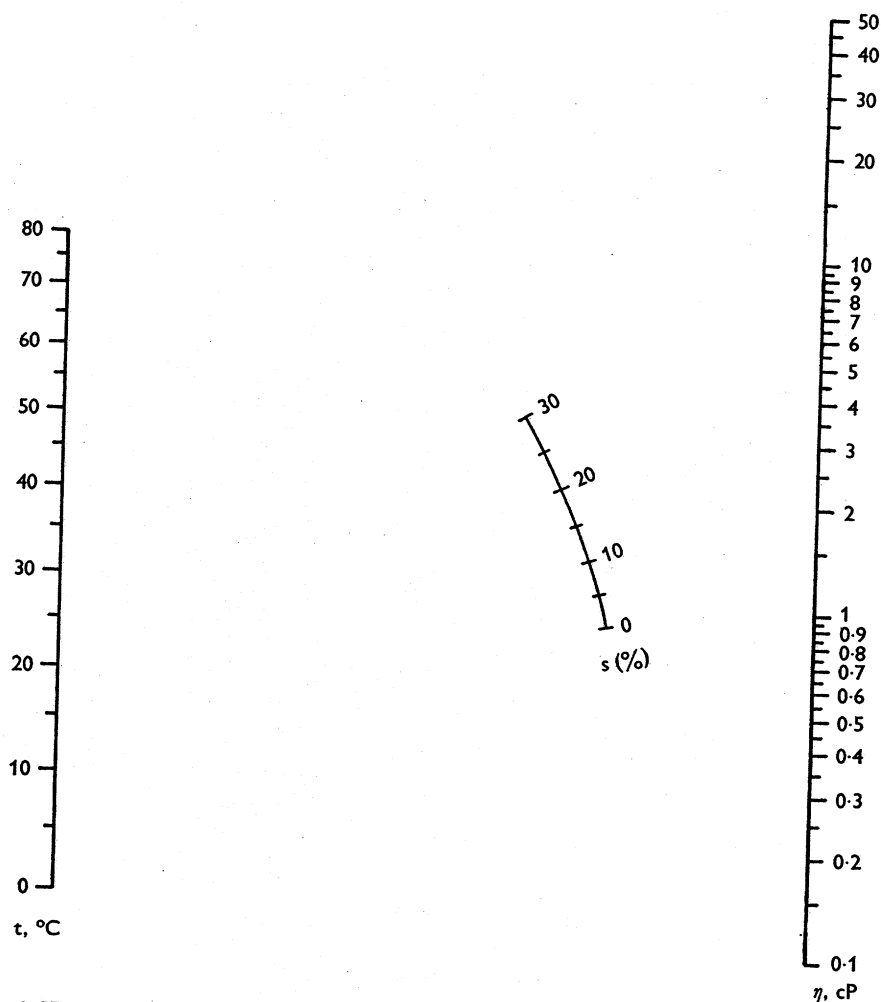


Fig. 6. Nomogram for the derivation of viscosity coefficients from temperature and total solids percentage for any S, H or W milk. η is obtained by drawing a straight line through the required t and s values.

From these regression formulae, empirical values for viscosity are graphically represented by full lines in Figs 1, 2 and 3; their deviations from the experimental data were generally about 1–2%.

Concentration effects are graphically shown in Fig. 4. Viscosity increased with total solids content according to a polynomial function which appeared to be independent of the fat:SNF ratio; the relationship could be expressed by the same mathematical model as that for temperature (Table 3).

Sample W-2.5 was omitted from this regression analysis since, being an evaporated milk, continuously sterilized in the can, its behaviour was not consistent with that of the other samples. In fact, this milk exhibited viscosity values about 60–70% higher than those predicted by the regression expressions of Table 3, while the fit of the experimental data for the remaining milks was generally within 5%. Empirical

viscosity-concentration curves shown by full lines in Fig. 4 are drawn in accordance with these equations.

By combining the 2 relationships discussed above it was possible to formulate a single expression interrelating viscosity, temperature and concentration as follows:

$$\log \eta = A_0 + A_1t + A_2t^2 + (B_0 + B_1t + B_2t^2)s + (C_0 + C_1t + C_2t^2)s^2 \quad (2)$$

whose coefficients A_i , B_i and C_i , calculated by the least squares method, are given in Table 4.

This general expression was found to fit the experimental data for any of the milks examined, W-2.5 excepted, usually to within 5 %, as can be seen in Fig. 5, where the theoretical curves and the experimental points can be compared. It is interesting to note that, when total solids content is made zero, the formula becomes a good fit for the viscosity values for water.

For a rapid evaluation of η , the general equation was graphically represented without practical loss of accuracy by a nomogram (Fig. 6) consisting of 2 parallel straight axes and one curved; values for the 3 interrelated quantities are connected by a straight-edge.

This work is a partial fulfilment of the Research Project No. UR-E25-(69)-37 financed in part by a grant made by the United States Department of Agriculture under P.L. 480.

REFERENCES

- CAFFYN, J. E. (1951). *J. Dairy Res.* **18**, 95.
COX, C. P. (1952). *J. Dairy Res.* **19**, 72.
FERNÁNDEZ-MARTÍN, F. (1972). *J. Dairy Res.* **39**, 65.
FERNÁNDEZ-MARTÍN, F. & MONTES, F. (1970). *18th Int. Dairy Congr., Sydney* **1E**, 471.
TORSSELL, H., SANDBERG, U. & THURESON, L.-E. (1949). *12th Int. Dairy Congr., Stockholm* **2**, 246.